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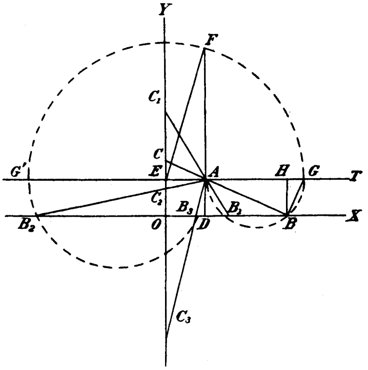
THE PROBLEM OF PAPPUS.

By Dr. H. A. Sayre.

Problem —Through a given point A in the bisector of a right angle to draw a secant of given length l.

The usual proof of the construction of Pappus may be modified so as to make it more symmetrical and elegant. For purposes of comparison the ordinary proof will first be given and then the modified proof.

Proof.*—Let us take as the unknown lines $DB = x$, $EC = y$, and put $AD = AE = a$, a being the distance from the point A to the sides of the right angle.



The similar triangles AEC, ADB give

$$xy = a^2. \tag{1}$$

We have, also,

$$(x + a)^2 + (y + a)^2 = l^2. \tag{2}$$

Eliminating y we obtain the equation

$$(x + a)^2 + \left[\frac{a^2}{x} + a\right]^2 = l^2,$$

which may be written

$$\left[\frac{x}{a} + 1\right]^2 + \left[\frac{a}{x} + 1\right]^2 = \frac{l^2}{a^2}.$$

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* Niewenglowski, Geometrie Analytique, Vol. I, p. 14. Pruvost, Geométrie Analytique, Vol. I, p. 18.

The last equation may be put in the form

$$\left[\frac{x}{a}+\frac{a}{x}\right]^2+2\left[\frac{x}{a}+\frac{a}{x}\right]-\frac{l^2}{a^2}=0\,.$$

Making the substitution

$$\frac{u}{a}=\frac{x}{a}+\frac{a}{x}\,;$$

we have finally

$$u^2+2au-l^2=0\,,$$

whence

$$u=-a\pm\sqrt{l^2+a^2}\,.$$

Take $AF=l$, so that $EF=\sqrt{a^2+l^2}$ and with E as a centre and EF as a radius, describe a circumference cutting the parallel to OX drawn through A in the points G and G'. Then

$$AG=\sqrt{l^2+a^2}-a\,,\quad AG'=\sqrt{l^2+a^2}+a\,.$$

Now

$$u=x+\frac{a^2}{x}\,.$$

If we draw BH perpendicular to AG, since $AH=x$, $HG=\frac{a^2}{x}$. It follows from this that ABG is a right angle. We describe therefore a semicircle upon AG as a diameter. If this circle cuts OX in B and B_1, the secants BC and B_1C_1 are solutions. We obtain in the same way two other solutions, B_2C_2 and B_3C_3.

Modified Proof.—Equation (2) may be written

$$(x+y)^2-2xy+2a(x+y)+2a^2=l^2\,.$$

Replacing xy by a^2 this equation becomes

$$(x+y)^2+2a(x+y)-l^2=0\,.$$

Solving we get

$$x+y=-a\pm\sqrt{l^2+a^2}\,. \tag{3}$$

Equations (1) and (3) show that the problem has been reduced to constructing two lines when their sum and geometrical mean are given. If we construct the sum of the two lines on the line AT we have the construction of Pappus.

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